CALCULATION OF THE INFLUENCE OF CENTRIFUGAL FORCES ON SURFACE PRESSURE FOR A BODY OF ARBITRARY SHAPE IN HYPERSONIC FLOW

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The fundamental geometrical property of flow around a body at hypersonic speeds is the proximity of the bow shock to the body surface (with a correspondingly large density in the layer behind the shock). This property has been used in a number of papers to set up approximate methods of calculation (cf. for example [1]). Particular attention has been paid to limiting cases in which the ratio of specific heats $\gamma \to 1$ and the Mach number $M \to \infty$ for a body of non-vanishing thickness. The shock then coincides with the body surface, and the density of the air behind the shock is infinitely large. Notwithstanding the fact that the thickness of the layer behind the shock is equal to zero in the limit, the pressure at the surface of the body will be different from the pressure immediately behind the shock owing to the action of centrifugal forces on the thin layer of high density. This effect of centrifugal forces on surface pressure was first computed by Busemann for two-dimensional and axially symmetric bodies, and Busemann's solution has been used as the first term in a series expansion in powers of a small parameter ($\epsilon = (\gamma - 1)/(\gamma + 1)$, $1/M_1^2$) [2, 3] and as a first approximation in a process of successive approximations [4, 5].

In all the papers mentioned the flows considered were two-dimensional, so that the calculations could be materially facilitated by the introduction of a stream function. In one paper [6] it was shown that the leading terms in an expansion of the stream function in powers of the distance from the shock also gives a satisfactory approximation. Although attempts have been made to estimate the effects of centrifugal forces on pressure for flow around a body of revolution at angle of attack [7], there exists no sufficiently reliable method of calculation, analogous to the Busemann method, for a body of arbitrary shape. An attempt to propose such a method is made in the present paper.

For the limiting case of a layer of vanishing thickness behind the bow shock, a method is presented for calculating surface pressure on a body of arbitrary shape. The streamlines in the layer behind the shock are determined as unaccelerated particle paths at the body surface as a function of the initial velocities. The streamline distribution through the layer is then found from the equation of continuity, and the pressure at the body surface is computed when the streamlines are known.

1. The equation of motion in a system of curvilinear coordinates based on the bow shock. It is well known that the surfaces parallel to a given surface S together with two families of developable surfaces generated by normals to the surface S in the directions of [principal] curvature make up an orthogonal system [8]*. We will take as surface S the bow shock, as curvilinear coordinates the parameters ξ, η on lines of curvature, and as coordinate ζ the normal distance measured from the shock surface toward the body.

Adopting the usual notation, in which E, G are the coefficients of the first fundamental quadratic form, L, N are the coefficients of the second fundamental quadratic form, and R_ξ, R_η are the principal radii of curvature [8], for the Lamé coefficients we will have**

$$H_\xi = \sqrt{E}\left(1-\frac{\zeta}{R_\xi}\right), \quad R_\xi = \frac{E}{L}, \quad \frac{\partial \ln H_\xi}{\partial \zeta} = \frac{1}{\zeta - R_\xi}, \quad H = 1$$
$$H_\eta = \sqrt{G}\left(1-\frac{\zeta}{R_\eta}\right), \quad R_\eta = \frac{G}{N}, \quad \frac{\partial \ln H_\eta}{\partial \zeta} = \frac{1}{\zeta - R_\eta},$$

In this system of coordinates the equations of Euler and the equations of continuity and isentropy for an ideal gas have the form

$$\frac{u}{H_\xi}\frac{\partial u}{\partial \xi} + \frac{v}{H_\eta}\frac{\partial u}{\partial \eta} + w\frac{\partial u}{\partial \zeta} + v\left(\frac{u}{H_\eta}\frac{\partial \ln H_\xi}{\partial \eta} - \frac{v}{H_\xi}\frac{\partial \ln H_\eta}{\partial \xi}\right) + uw\frac{\partial \ln H_\xi}{\partial \zeta} = -\frac{1}{\rho H_\xi}\frac{\partial p}{\partial \xi} \tag{1.1}$$
$$\frac{u}{H_\xi}\frac{\partial v}{\partial \xi} + \frac{v}{H_\eta}\frac{\partial v}{\partial \eta} + w\frac{\partial v}{\partial \zeta} - u\left(\frac{u}{H_\eta}\frac{\partial \ln H_\xi}{\partial \eta} - \frac{v}{H_\xi}\frac{\partial \ln H_\eta}{\partial \xi}\right) + vw\frac{\partial \ln H_\eta}{\partial \zeta} = -\frac{1}{\rho H_\eta}\frac{\partial p}{\partial \eta}$$

* V.V. Struminskii has used such a system for the derivation of the boundary layer equations.

** The index denotes the quantity varying along the coordinate lines.

$$\frac{u}{H_\xi}\frac{\partial w}{\partial \xi} + \frac{v}{H_\eta}\frac{\partial w}{\partial \eta} + w\frac{\partial w}{\partial \zeta} - u^2\frac{\partial \ln H_\xi}{\partial \zeta} - v^2\frac{\partial \ln H_\eta}{\partial \zeta} = -\frac{1}{\rho}\frac{\partial p}{\partial \zeta}$$

$$\frac{\partial \rho u H_\eta}{\partial \xi} + \frac{\partial \rho v H_\xi}{\partial \eta} + \frac{\partial \rho w H_\xi H_\eta}{\partial \zeta} = 0, \qquad \frac{u}{H_\xi}\frac{\partial}{\partial \xi}\left(\frac{p}{\rho^\gamma}\right) + \frac{v}{H_\eta}\frac{\partial}{\partial \eta}\left(\frac{p}{\rho^\gamma}\right) + w\frac{\partial}{\partial \zeta}\left(\frac{p}{\rho^\gamma}\right) = 0$$

where u, v, w are the velocity components in the direction of the coordinate lines. Equations (1.1) represent a system of five quasilinear equations in five unknowns u, v, w, p, ρ. From these five equations may be determined five first derivatives with respect to ζ at $\zeta = 0$, using given data immediately behind the shock. The derivatives $\partial u/\partial \zeta$, $\partial v/\partial \zeta$ are computed directly from the first two of equations (1.1). The determinant of the remaining three equations, from which the derivatives of w, p, and ρ with respect to ζ are to be found, is

$$\begin{vmatrix} w & \frac{1}{\rho} & 0 \\ \rho & 0 & w \\ 0 & w\rho^{-\gamma} & -\gamma p w \rho^{-(\gamma+1)} \end{vmatrix} = \frac{w}{\rho^\gamma}\left(\gamma\frac{p}{\rho} - w^2\right)$$

and reduces to zero only if the shock degenerates into a weak wave. Further derivatives with respect to ζ may be found by differentiating equations (1.1) with respect to ζ, whereas the determinant of the last three equations will be the same.

2. Velocity, density, and pressure immediately behind the bow shock. The normal velocity component w_2, the density ρ_2, and the pressure p_2 immediately behind the shock are expressed in terms on the normal velocity component w_1, the density ρ_1, and the pressure p_1 ahead of the shock in the following way;

$$w_2 = w_1\left(\varepsilon + \frac{1-\varepsilon}{w_1^2}a_1^2\right), \qquad \frac{\rho_1}{\rho_2} = \varepsilon + (1-\varepsilon)\frac{a_1^2}{w_1^2} \tag{2.1}$$

$$p_2 = (1-\varepsilon)\rho_1 w_1^2\left(1 - \frac{\varepsilon}{1+\varepsilon}\frac{a_1^2}{w_1^2}\right) \quad (a_1 = \text{velocity of sound})$$

Note: If the enthalpy behind the shock may be taken as

$$i_2 = \frac{\gamma_2}{\gamma_2 - 1}\frac{p_2}{\rho_2}$$

but $\gamma_2 \neq \gamma_1$ because of the high temperature, then

$$w_2 = \frac{w_1\gamma_2}{\gamma_2+1}\left[1 + \frac{a_1^2}{\gamma_1 w_1^2} - \left(\frac{1}{\gamma_1^2}\left(1 - \frac{a_1^2}{w_1^2}\right)^2 + \left(\frac{1}{\gamma_2^2} - \frac{1}{\gamma_1^2}\right)\left[1 + \frac{2a_1^2}{(\gamma_1-1)w_1^2}\right]\right)^{1/2}\right]$$

so that for $\gamma_2 \to 1$ the velocity w_2 behind the shock approaches zero independently of w_1/a_1.

If we assume that the vector velocity V in the undisturbed stream (ahead of the shock) is situated in the xy plane and makes an angle α with the x-axis, then the components of this velocity in the curvilinear coordinate system will be

$$w_1 = \frac{V}{\sqrt{EG}}\left[\cos\alpha\,\frac{\partial(y,z)}{\partial(\xi,\eta)} + \sin\alpha\,\frac{\partial(z,x)}{\partial(\xi,\eta)}\right] \tag{2.2}$$

$$u_1 = u_2 = \frac{V}{\sqrt{E}}\left(\cos\alpha\,\frac{\partial x}{\partial\xi} + \sin\alpha\,\frac{\partial y}{\partial\xi}\right), \quad v_1 = v_2 = \frac{V}{\sqrt{G}}\left(\cos\alpha\,\frac{\partial x}{\partial\eta} + \sin\alpha\,\frac{\partial y}{\partial\eta}\right)$$

In order for it to be possible to introduce a stream function, one of the derivatives in the continuity equation (the fourth of equations 1.1), for example $\partial\rho v H_\xi/\partial\eta$, has to be put equal to zero, in particular, immediately behind the shock. This is possible only if all the derivatives of the coordinates x, y, z with respect to ξ and η are functions only of ξ at the shock, or if $\cos\alpha\,\partial x/\partial\eta + \sin\alpha\,\partial y/\partial\eta = 0$ at the shock.

In the first case the shock is a cylindrical surface, and the lines η [= constant] are straight lines; in the second case the shock is an axially symmetric or helical surface, and the lines η [= constant] are either circles or straight lines. The shock configurations just specified exhaust the cases for which a stream function may be introduced.

3. Approximate equations for the case of a "strong" shock. Equations (1.1) are materially simplified if the density behind the shock is much greater than the density in the undisturbed stream (the limit $\rho_2/\rho_1 \to \infty$). This occurs for $\epsilon \to 0$ or for $a_1^2/w_1^2 \to 0$; i.e. for $M_1 \to \infty$, provided that the angle between the normal to the shock surface and the vector velocity V does not approach $\pi/2$ (nonvanishing body thickness). This case corresponds to a value of the similarity criterion for supersonic velocities $K \to \infty$. For the conditions specified it may be assumed that quantities are of the same order in the layer behind the shock as they are just behind the shock, namely

$$w = \varepsilon V w', \quad u = Vu', \quad v = Vv', \quad \frac{\rho_1}{\rho} = \frac{\varepsilon}{\rho'} \quad p = \rho_1 V^2 p'$$

where ϵ is a small parameter and w', u', v', p', ρ' are quantities of order unity. The coordinate ζ is of the order of the small parameter ϵ compared to the two other coordinates. The elimination of quantities of order ϵ or higher in equations (1.1) compared to quantities of order unity gives the following simplified equations:

$$\frac{u}{\sqrt{E}}\frac{\partial u}{\partial\xi} + \frac{v}{\sqrt{G}}\frac{\partial u}{\partial\eta} + w\frac{\partial u}{\partial\zeta} + v\left(\frac{u}{\sqrt{G}}\frac{\partial\ln\sqrt{E}}{\partial\eta} - \frac{v}{\sqrt{E}}\frac{\partial\ln\sqrt{G}}{\partial\xi}\right) = 0 \tag{3.1}$$

$$\frac{u}{\sqrt{E}}\frac{\partial v}{\partial \xi}+\frac{v}{\sqrt{G}}\frac{\partial v}{\partial \eta}+w\frac{\partial v}{\partial \zeta}-u\left(\frac{u}{\sqrt{G}}\frac{\partial \ln \sqrt{E}}{\partial \eta}-\frac{v}{\sqrt{E}}\frac{\partial \ln \sqrt{G}}{\partial \xi}\right)=0 \tag{3.2}$$

$$\frac{u^2}{R_\xi}+\frac{v^2}{R_\eta}=-\frac{1}{\rho}\frac{\partial p}{\partial \zeta} \tag{3.3}$$

$$\frac{\partial \rho u\sqrt{G}}{\partial \xi}+\frac{\partial \rho v\sqrt{E}}{\partial \eta}+\frac{\partial \rho w\sqrt{GE}}{\partial \zeta}=0 \tag{3.4}$$

$$\frac{u}{\sqrt{E}}\frac{\partial}{\partial \xi}\left(\frac{p}{\rho}\right)+\frac{v}{\sqrt{G}}\frac{\partial}{\partial \eta}\left(\frac{p}{\rho}\right)+w\frac{\partial}{\partial \zeta}\left(\frac{p}{\rho}\right)=0 \tag{3.5}$$

Multiplying the first equation by u and the second by v and adding, we have

$$\frac{u}{\sqrt{E}}\frac{\partial (u^2+v^2)}{\partial \xi}+\frac{v}{\sqrt{G}}\frac{\partial (u^2+v^2)}{\partial \eta}+w\frac{\partial (u^2+v^2)}{\partial \zeta}=0 \tag{3.6}$$

That is, the projection of the velocity on a plane tangent to the shock does not change along a streamline. Thus in the case of high density but finite pressure behind the shock the gas in the thin layer between the shock and the body moves without acceleration in a tangential plane, while the component of acceleration normal to the shock (body) surface is balanced by a pressure gradient normal to the shock surface. We will use the notation $\operatorname{tg}\theta = u/v$, $q = \sqrt{u^2+v^2}$; then, taking account of (3.6), instead of equations (3.1)-(3.5) we obtain

$$\frac{\cos\theta}{\sqrt{E}}\frac{\partial \theta}{\partial \xi}+\frac{\sin\theta}{\sqrt{G}}\frac{\partial \theta}{\partial \eta}+\frac{w}{q}\frac{\partial \theta}{\partial \zeta}=\frac{\cos\theta}{\sqrt{G}}\frac{\partial \ln\sqrt{E}}{\partial \eta}-\frac{\sin\theta}{\sqrt{E}}\frac{\partial \ln \sqrt{G}}{\partial \xi} \tag{3.7}$$

$$\frac{\cos^2\theta}{R_\xi}+\frac{\sin^2\theta}{R_\eta}=-\frac{1}{\rho q^2}\frac{\partial p}{\partial \zeta} \tag{3.8}$$

$$\sqrt{EG}\frac{\partial}{\partial \zeta}\left(\frac{\rho w}{q}\right)+\frac{\partial \rho\cos\theta\sqrt{G}}{\partial \xi}+\frac{\partial \rho\sin\theta\sqrt{E}}{\partial \eta}=0 \tag{3.9}$$

$$\frac{\cos\theta}{\sqrt{E}}\frac{\partial}{\partial \xi}\left(\frac{p}{\rho}\right)+\frac{\sin\theta}{\sqrt{G}}\frac{\partial}{\partial \eta}\left(\frac{p}{\rho}\right)+\frac{w}{q}\frac{\partial}{\partial \zeta}\left(\frac{p}{\rho}\right)=0 \tag{3.10}$$

The assumptions adopted here concerning the small thickness of the layer behind the shock and the high density in this layer are permissible as long as the pressure at the surface of the body does not become small ($p \approx 0$), in which case there is a corresponding decrease in the density and the shock leaves the body. Under these conditions the disturbances are already small, and it is not permissible in the equations of motion to discard quantities of order ϵ. An estimate of the order of the terms in the equations of motion for this case of small disturbances ($K > 0$, but $K \neq \infty$) leads to certain simplifications [1]; however, the use of a system of coordinates based on the shock is unsuitable for this case*.

4. Approximate solution. One method for computing the flow behind the shock is the use of series in powers of ζ, the coefficients of which are calculated from equations (1.1)-(1.5) or (3.1)-(3.5).

The accuracy of such calculations (series convergence), especially for the region of subsonic flow in front of a blunt-nosed body, needs investigation; nevertheless, the results of sufficiently rigorous calculations [9] for a body of revolution ($M = 5.8$) apparently show that in the subsonic region the change in the velocity components with a change in ζ may be adequately represented by the first terms of the series [6]. For pressure and density it is better to use the Bernoulli equation

$$\frac{u^2+v^2+w^2}{2} + \frac{1+\varepsilon}{2\varepsilon}\frac{p}{\rho} = \frac{V^2}{2} + \frac{1+\varepsilon}{2\varepsilon}\frac{p_1}{\rho_1}$$

With the object of confirming this explicit assumption, we will present the results of calculations, using the series, of the distance Δ between the shock and the stagnation point of a blunt axially symmetric body. From equations (3.1)-(3.5), on the longitudinal body axis behind the shock we obtain

$$\left(\frac{\partial w}{\partial \zeta}\right)_2 = -\frac{2V}{R_c}, \qquad \left(\frac{\partial^2 w}{\partial \zeta^2}\right)_2 = \frac{V^2}{w_2 R^2_c}$$

where R_c is the radius of curvature of the shock. Consequently

$$w = w_2 - \frac{2V\zeta}{R_c} + \frac{V^2\zeta^2}{2w_2R^2_c} + \ldots$$

At the stagnation point of the body $w = 0$, so that for the distance between the shock and the stagnation point we obtain

$$\Delta = (2-\sqrt{2})\frac{w_2}{V}R_c, \qquad \frac{\Delta}{R_c} = (2-\sqrt{2})\varepsilon \qquad \text{for } M_1 \to \infty$$

This value of Δ for $M_1 \to \infty$ proves to be close to the value calculated by the method of Ref. 6. The use of series for the case of a body of arbitrary shape will lead to a complicated computation, and here we will employ another procedure suitable for the solution of equations (3.7)-(3.10), given the normal velocity, density, and pressure at the shock in accordance with equations (2.1)-(2.3).

* It is an interesting circumstance that in the case of a thin wing lying in the xz plane the equations of motion for small disturbances reduce to the non-steady equations of motion for a piston at each cross-section whose plane is parallel to the plane xy.

Considering the extreme thinness of the layer between the shock and the body, we will assume that the particle paths in the gas are [equally] inclined to the surface of the shock; then in place of (3.7) we will obtain

$$\frac{\cos\theta}{\sqrt{E}}\frac{\partial\theta}{\partial\xi}+\frac{\sin\theta}{\sqrt{G}}\frac{\partial\theta}{\partial\eta}=\frac{\cos\theta}{\sqrt{G}}\frac{\partial\ln\sqrt{E}}{\partial\eta}-\frac{\sin\theta}{\sqrt{E}}\frac{\partial\ln\sqrt{G}}{\partial\xi} \tag{4.1}$$

This assumption may give rise to appreciable errors only in the neighborhood of the stagnation point of a blunt body. If the equation of a streamline is $\eta = \eta(\xi)$, then $\operatorname{tg}\theta = \sqrt{G/E}\,d\eta/d\xi$. But the left-hand side of equation (4.1) is nothing but the derivative of the angle θ along the arc of a streamline; i.e. $(\partial\theta/\partial\xi)\cos\theta/\sqrt{E}$. Consequently, in place of (4.1) we will have

$$\frac{d\theta}{d\xi}=\frac{1}{\sqrt{G}}\left(\frac{\partial\sqrt{E}}{\partial\eta}-\operatorname{tg}\theta\,\frac{\partial\sqrt{G}}{\partial\xi}\right) \tag{4.2}$$

From this equation we obtain an ordinary nonlinear second-order equation to determine the streamlines;

$$\frac{d^2\eta}{d\xi^2}+\frac{1}{2E}\frac{\partial G}{\partial\xi}\left(\frac{d\eta}{\partial\xi}\right)^3+\frac{\partial}{\partial\eta}\ln\left(\frac{\sqrt{G}}{E}\right)\left(\frac{d\eta}{d\xi}\right)^2+\frac{\partial}{\partial\xi}\ln\left(\frac{G}{\sqrt{E}}\right)\frac{d\eta}{d\xi}-\frac{1}{2G}\frac{\partial E}{\partial\eta}=0 \tag{4.3}$$

Having the projection of the streamlines on the shock surface, it is now necessary to distribute these streamlines across the layer in such a way as to satisfy the continuity equation.

At any point (ξ, η) the streamlines passing through a normal to the shock surface end on some line L in the shock surface. We will take a surface element defined by an element of this line $L(d\sigma_2)$ and an element of its normal (dn_2) at a point (ξ_2, η_2). The equation of continuity for a stream tube passing through this surface element will be $\rho_1 w_1 d\sigma_2 dn_2 = \rho\, q\, d\zeta\, dn$, where dn is the distance between the streamlines generating the stream tube (passing through the ends of the segment dn_2).

Let the equation of a streamline be given in the form $\eta = \eta(\xi_2, \eta_2, \xi)$. The increment in the coordinate η for a change in the parameters ξ_2, η_2 together with a displacement along the stream line should be equal to the increment in the coordinate η for a displacement along a normal to the streamline at the point (ξ, η);

$$d\eta=\frac{\partial\eta}{\partial\xi_2}d\xi_2+\frac{\partial\eta}{\partial\eta_2}d\eta_2+\frac{d\eta}{d\xi}d\xi=-\sqrt{\frac{E}{G}}\operatorname{ctg}\theta d\xi.$$

From this

$$d\xi=-\frac{\frac{\partial\eta}{\partial\xi_2}d\xi_2+\frac{\partial\eta}{\partial\eta_2}\partial\eta_2}{\frac{\partial\eta}{\partial\xi}+\sqrt{\frac{E}{G}}\operatorname{ctg}\theta}=-\sqrt{\frac{G}{E}}\left(\frac{\partial\eta}{\partial\xi_2}d\xi_2+\frac{\partial\eta}{\partial\eta_2}d\eta_2\right)\cos\theta\sin\theta$$

The increments in the parameters ξ_2, η_2 are found from the quantity dn_2, and we obtain

$$-\sqrt{E_2}d\xi_2 = dn_2 \sin\gamma_2, \qquad \sqrt{G_2}d\eta_2 = dn_2 \cos\gamma_2$$

where γ_2 is the angle between the tangents to the line L and to the coordinate line ξ at the point (ξ_2, η_2). Inasmuch as $d\xi = -(1/\sqrt{E}) \sin\theta\, dn$, then

$$dn = dn_2 \cos\theta \sqrt{G}\left(\frac{\cos\gamma_2}{\sqrt{G_2}}\frac{\partial\eta}{\partial\eta_2} - \frac{\sin\gamma_2}{\sqrt{E_2}}\frac{\partial\eta}{\partial\xi_2}\right) = m(\sigma_2)\, dn_2$$

Substituting this expression in the continuity equation, we obtain

$$d\zeta = \frac{\rho_1 w_1 d\sigma_2}{\rho q m(\sigma_2)} \tag{4.4}$$

The variable of integration in (3.8) may now be changed from ζ to σ_2:

$$\frac{\partial p}{\partial\sigma_2} = -\frac{\rho_1 w_1 q}{m(\sigma_2)}\left(\frac{\cos^2\theta}{R_\xi} + \frac{\sin^2\theta}{R_\eta}\right) \tag{4.5}$$

The quantities entering into (4.5) depend both on the coordinates of the point (ξ, η) as parameters and on the coordinates of the point (ξ_2, η_2) on the line L along which the integration is to be performed. Integration of (4.5) may be carried out from a point (ξ, η) in the shock surface to any point on the line L; the corresponding values of ζ may then be determined from (4.4), including the point lying in the body surface. The formula (4.5) is a generalization, for a body of arbitrary shape, of the Busemann formula.

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